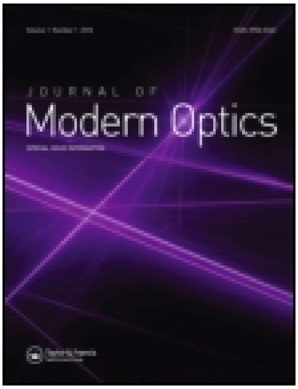

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Lateral Support of Very Large Telescope Mirrors by Edge Forces Only

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Lateral support of very large telescope mirrors by edge forces only

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Abstract. Until now a diameter of about 4 m seemed to be the upper-size limit of telescope mirrors that still permitted cost-saving designs of lateral supports by edge forces alone. In some designs, the supporting edge forces comprised not only basic push–pull action normal to the edge but also a specific, moderate amount of tangential shear. However, this was a by-product of design economy rather than the result of understanding the potential of tangential support forces as a means of reducing mirror flexure systematically, down to residuals in the 1% region. The surprising possibility of extending the usefulness of pure edge supports is demonstrated by the example of the 8-m mirror of the European Southern Observatory's Very Large Telescope. Fitted with a lateral support at the outer edge alone, this thin mirror will exhibit a wavefront aberration with a calculated rms value of only 18 nm, without taking into account possible active control.

1. Introduction

During the last few decades, a number of telescope mirrors with diameters up to about 4 m, and of more or less conventional aspect ratios, were fitted with lateral supports in which the supporting forces were exerted only at the outer edge in the form now known as push–pull support. It is characterized by an angular distribution of peripheral forces normal to the cylindrical edge, i.e. perpendicular to the optical axis, and following a cosine law with respect to the azimuth angle. The underlying principle had been suggested and analysed many years ago [1] and, thereafter, been repeatedly investigated, either in a general form or in conjunction with particular telescope projects [2–6]. Push–pull supports are simpler than, for instance, lateral supports, which are internally nested through the back face. They also are more easily adjustable. However, they could only be successfully put into practice after progress in bonding technology permitted safe application of tensile supporting forces.

The push–pull principle was later extended to include an additional lift by a distribution of tangential shearing forces at the outer edge. Such supports could be designated push–pull–shear supports. It will be shown that the systematic utilization of the shear component, whose potential had not been fully recognized until now, can lead to surprising improvements. The bending distortion of large mirrors can be reduced drastically; in fact, so much that the practical application of such support designs need no longer be effectively limited by mirror size.

2. Tangential shearing forces as a design tool

Let the elastic surface deflection w of a circular solid mirror in its upright position under the action of gravity be expressed in the form of a Fourier expansion:

$$w(r,\theta)=\sum_{n=1}^{\infty} w_n(r)\cos n\theta, \qquad (1)$$

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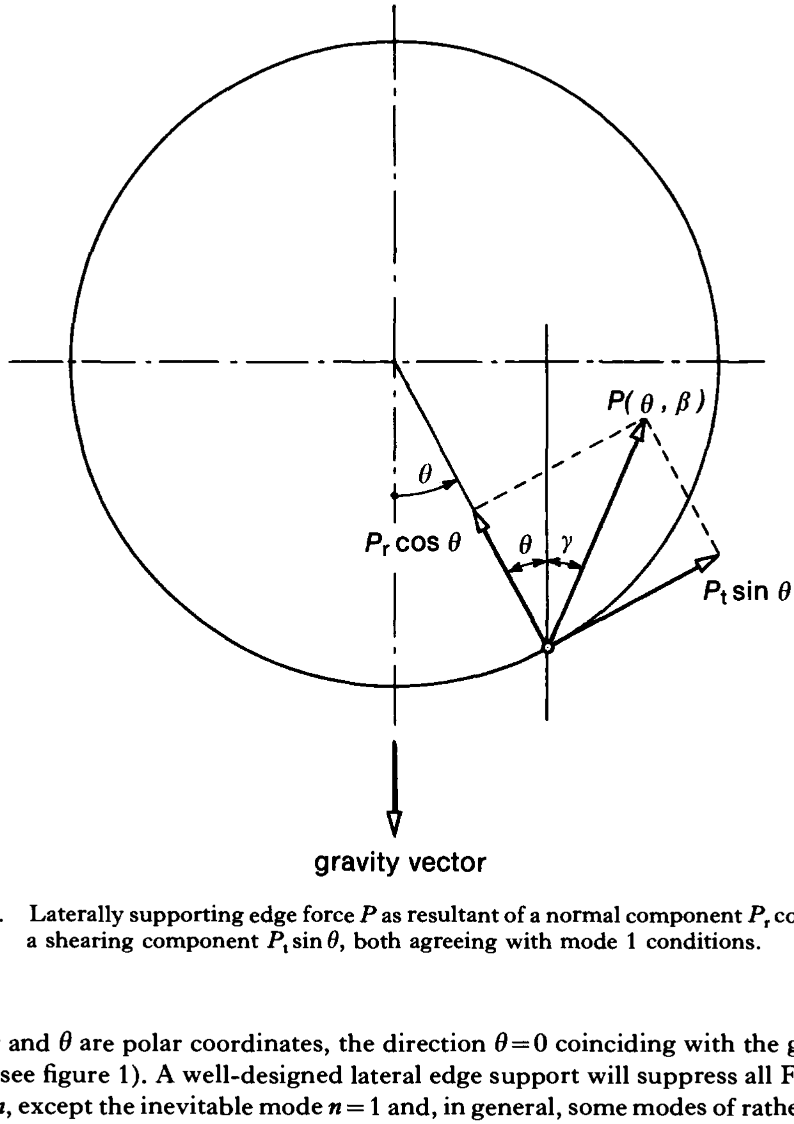


Figure 1. Laterally supporting edge force P as resultant of a normal component $P_r \cos\theta$ and a shearing component $P_t \sin\theta$, both agreeing with mode 1 conditions.

where r and θ are polar coordinates, the direction $\theta = 0$ coinciding with the gravity vector (see figure 1). A well-designed lateral edge support will suppress all Fourier modes n, except the inevitable mode $n = 1$ and, in general, some modes of rather high orders of no practical significance. The lowest of these is smaller by 1 than the number of equidistant radial support points on the circumference of the mirror. Thus, in practice, the limit of applicability of push–pull edge supports with respect to elastic flexure is determined solely by the shape and magnitude of the mode $n = 1$. Both result from the interplay of the support parameters chosen to maintain mirror equilibrium. In push–pull–shear supports it is the particular balance between the normal edge force $P_r \cos\theta$ and the tangential shearing force $P_t \sin\theta$ (figure 1) that most strongly influences the magnitude of mode 1. The forces P_r and P_t, assumed positive in the sense shown (the usual sign convention for P_r is opposite, however), act in a plane normal to the optical axis and bisecting the mirror edge.

Push–pull supports in which part of the mirror weight is supported at the central hole have in a few cases been investigated [4, 6], but their advantages may still be doubted. If some reduction of flexure can be achieved, it must be paid for by higher orders of coma aberration that are not easily correctable. Such appearance of higher coma orders will be noticed in an example treated later (see figure 6). A further distinct disadvantage is the technical difficulty of fitting a rather sophisticated support system into the relatively small mirror hole while at the same time retaining sufficient space for necessary telescope instrumentation.

Let the symbol β denote the fraction of the weight supported by tangential edge forces relative to the total weight support at the same edge. One can then immediately conclude from figure 1, after integrating the upwardly directed force components over the full circumference, that

$$\beta = P_t/(P_t + P_r). \tag{2}$$

In the first genuine push–pull supports applied in telescope design the mirror weight was laterally supported only by normal edge forces in a cosine distribution; for example, the 4-m KPNO, 2·2-m MPIA, 3·5-m MPIA telescopes, which all represent the special case $\beta = 0$.

In the general case of non-zero values of β the radial force and the tangential force add up to a vector $\mathbf{P}(\theta, \beta)$, which is rotated clockwise by an angle γ with respect to the vertical. The vectorial addition of the edge forces yields a resultant of the magnitude

$$\mathbf{P} = \frac{P_r}{1-\beta}[(1-2\beta)\cos^2\theta + \beta^2]^{1/2}. \tag{3}$$

The clockwise rotation of the resultant vector is determined by

$$\tan\gamma = \frac{2\beta - 1}{(1-\beta)\cot\theta + \beta\tan\theta}. \tag{4}$$

If $\beta = 0$, the last two equations reduce to

$$\mathbf{P} = P_r \cos\theta, \tag{5}$$

$$\gamma = -\theta. \tag{6}$$

These are in fact the well-known conditions for the original push–pull supports mentioned above. Putting $\beta = 0{\cdot}5$, we find

$$\mathbf{P} = P_r = \text{const.} \tag{7}$$

$$\gamma = 0. \tag{8}$$

These conditions form the basis of support designs that are being used with telescope mountings of the alt–azimuth type. In this case they become particularly simple. There also existed, a now obsolete, more complex version [7] that was conceived for equatorial mounts, shortly before the introduction of modern alt–azimuth mounts. But there are at least two large telescopes that took advantage of the simple design with $\beta = 0{\cdot}5$; namely, the Anglo–Dutch 4·2-m Herschel telescope [8] and ESO's 3·5-m New Technology Telescope [9]. The Herschel telescope does, however, not belong to the pure push–pull–shear type with $\beta = 0{\cdot}5$, because the flexure of its mirror also involves the Fourier mode 3. A full account is given in [9].

Values of β other than the two special ones considered above do not seem to have received particular attention so far. It will be shown that these other values, namely values greater than 0·5, open up surprising possibilities. An example with $\beta = 0{\cdot}8$, briefly discussed in [9], was however a singular case, only meant to illustrate a possibility of suppressing, in part, higher aberration orders in mode. 1. Thus, it does not directly concern the essential point that will later be brought to light in connection with β-values exceeding 0·5. In fact, it turns out that the value 0·8 already lies somewhat beyond the useful range.

No attention will be paid either to β-values below 0·5; from exploratory trials, though more evidence is lacking, it is surmised that such values do not offer clear advantages. The analysis will therefore be restricted to the range $\beta > 0{\cdot}5$. As an example, figure 2 illustrates (for the arbitrary value $\beta = 0{\cdot}68$) the way in which the lateral edge forces are modified, compared with the standard case according to equations (7) and (8).

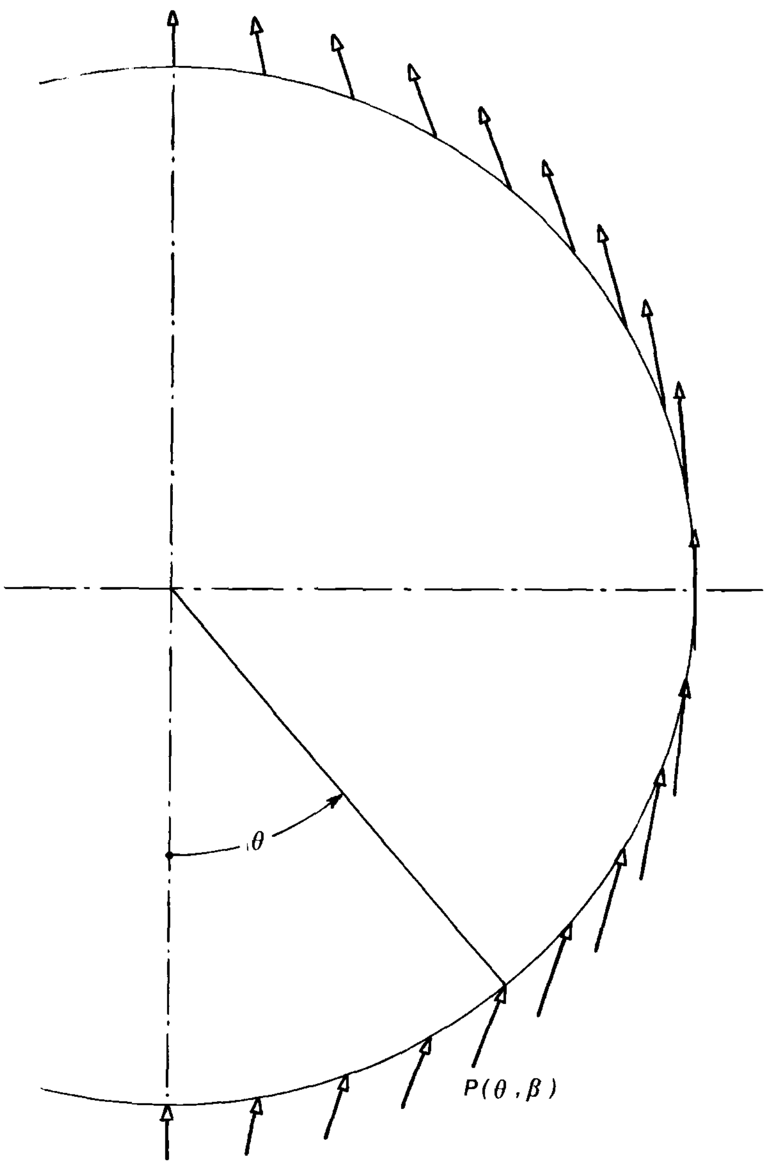


Figure 2. Lateral edge forces **P** corresponding to a shear fraction $\beta = 0{\cdot}680$.

3. Application to an 8-m mirror

The object to be investigated will be a lateral support for the 8-m mirror of the Very Large Telescope (VLT) of ESO. This mirror is a thin monolithic meniscus of Zerodur ceramics with the following principal dimensions:

outer diameter 8200 mm,
hole diameter 1000 mm,
thickness 175 mm,
radius of curvature of the middle surface 28888 mm.
f-number corresponding to outer diameter 1·76

With its very small aspect ratio of 1 : 47 and small *f*-number, the VLT primary is a sensitive gauge for exploring the possibility of lateral support by no other means that pure edge forces for a very large mirror. The decision to initiate such an investigation was made by ESO in the course of the support design there. Some previous considerations pursued by the author had indicated that an investigation of the problem would probably unveil surprises regarding the influence of the parameter β.

The design of ESO originally provided 48 equidistant support units at the outer mid-edge and 10 at the inner one. These units produced lateral support forces corresponding to the parameter value $\beta = 0{\cdot}5$. The outer supports also provided axial forces with a cosine distribution which maintained the meniscus in the static equilibrium. Its bending distortion was to be corrected by 150 actuators on its back face. Finite element calculations showed that the rms value of the residual wave aberration could be reduced to 14 nm, but only by using considerable axial correction forces up to about 200 N.

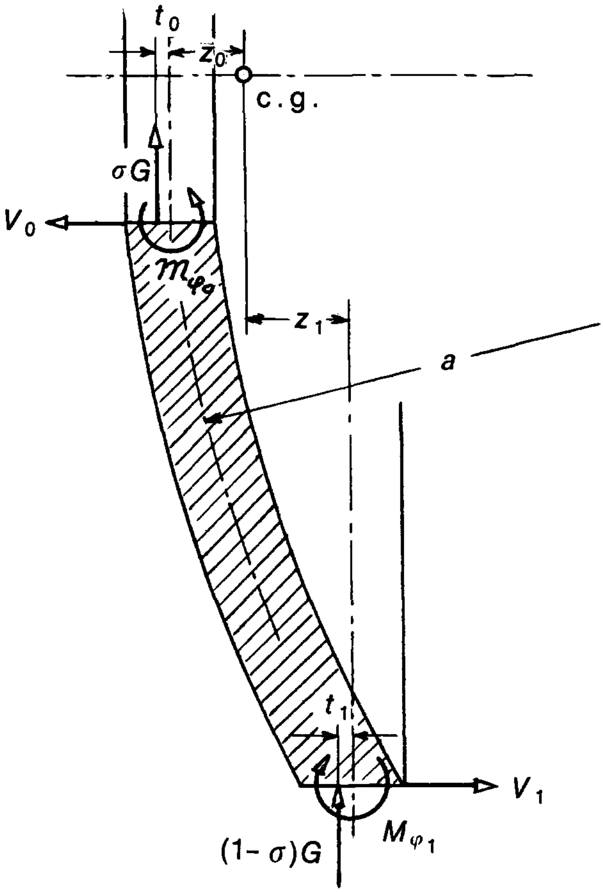
An rms value of 50 nm was set as the goal of the author's attempt to derive an analytical solution for a pure edge support, which would, hopefully, obviate extensive active correction and thus preserve the full dynamic range of the active control system for other corrections. To reach this goal, there were several optional support parameters available, namely, axial forces in cosine distributions at both edges and, within narrow limits, also bending moments at both edges, again in cosine distributions. Such moments arise if the radial edge force resultants do not pass through the mid-edge. Further parameters entering into the analysis are the fraction of the mirror weight to be supported at the inner edge and the two β-ratios at which tangential and radial forces act together at the two edges. Altogether there are seven parameters (see figure 3). The five parameters σ, ε_0, ε_1, τ_0 and τ_1 are not entirely independent of each other but must satisfy the following equilibrium condition for the moments

$$\varepsilon_0 - \varepsilon_1 + \sigma(\zeta_0 + \tau_0) - (1-\sigma)(\zeta_1 - \tau_1) = 0. \qquad (9)$$

The present analysis uses the theory developed in [9]. However, as several parameters previously not considered now play a part, different systems of boundary conditions had to be formulated.

4. Results for two systems of boundary conditions

From the many combinations of support parameters investigated, only those two will be discussed which gave the best and second-best result, the latter already four times worse. All other combinations can be dismissed because they only tend to degrade the first ranking result or, still worse, were inferior from the outset.



a Radius of curvature of the middle surface of the shell (28887·5 mm).
G Mass of mirror meniscus (23240 kg).
$i=0$ Index designating quantities at the inner edge (central hole).
$i=1$ Index designating quantities at the outer edge.
$M_{\varphi i}$ Radial bending moment at the edge.
R_i Radius of the cylindrical edge ($R_0=500$ mm, $R_1=4100$ mm).
t_i Spacing between mid-edge and plane of support force resultant (see figure).
V_i Axial shearing force at the edge.
z_i Distance of centre of gravity from mid-edge.
β_i Relative contribution of tangential shearing forces to the support of weight at the respective edge.
$1-\beta_i$ Corresponding relative contribution of radial forces (normal forces).
ε_i Fraction of the moment Ga exerted by axial edge forces $V_i \cos\theta$.
$\zeta_i = z_i/a$ ($\zeta_0 = 4{\cdot}98678 \times 10^{-3}$, $\zeta_1 = 4{\cdot}98670 \times 10^{-3}$).
σ Fraction of mirror weight supported at the hole.
$\tau_i = t_i/a$

Figure 3. Notation for the geometry of the mirror meniscus and the applied edge forces.

It was found that the choice of the parameter β_1 (defined in figure 3) plays a key role, inasmuch as other attempts to derive acceptable solutions were futile unless β_1 was well above 0·5. In other words, an edge force distribution similar to the one shown in figure 2 is a prerequisite to a successful result. In this respect, the other parameters are of little or no help. Attempts to obtain improvements by choosing other parameter combinations seem to do more harm than good.

4.1. *Mirror balance by axial forces at the outer edge*

The first and most important case is marked by zero values of σ, ε_0, τ_0 and τ_1.

The only parameter to be varied freely is β_1. Equation (9) then reduces to the obvious condition $\varepsilon_1 = -\zeta_1$. It requires that equilibrium be maintained only by axial forces at the outer edge. This system of boundary conditions was expressed in the form

$$M_{\varphi_0} = 0,$$
$$V_0 = 0,$$
$$P_{t_0} = 0,$$
$$M_{\varphi_1} = 0,$$
$$P_{t_1}/P_{r_1} = \beta_1/(1-\beta_1).$$

First it will be shown what happens if we insert the conventional value $\beta_1 = 0{\cdot}5$. The corresponding deflection w for $\theta = 0$, i.e. the radial profile $w_1(r)$ of mode 1 according to equation (1), is plotted in figure 4. It reaches an rms value of about 4000 nm, an amount totally out of the question. However, by a straight increase of β_1 to values near 0·75 the deflection drops drastically by two orders of magnitude, as shown in figure 5.

With the above boundary conditions and varying only β_1, the distortion of the mirror surface retains its intrinsic general shape, apart from a possible sign reversal, while the deflection ordinates may undergo changes by order of magnitude, as seen by comparing figures 4 and 5. Consequently, when β_1 is increased to a certain value, the surface distortion vanishes almost entirely and change of sign occurs. This meniscus flexure behaviour opens up the possibility of using only a lateral support at the outer edge, with negligible loss of optical quality, for an 8-m mirror. For $\beta_1 = 0{\cdot}7529$, which is the most favourable value in the present case, the analytical solution leads to wavefront aberrations with an rms value of 18 nm, an amazingly small amount in view of the large mirror diameter. This result is in line with later finite element calculations performed at ESO. With tangential shearing forces corresponding to the analytical result and with small additional correction forces of less than 10 N (necessary only to compensate an undesired extraneous effect†), the finite element evaluation revealed an rms value of 6 nm for the residual wavefront error.

† With $\beta_1 = 0{\cdot}7529$, equations (2) and (3) yield a support force P varying between the extremes P_r and P_t in a ratio 1 : 3·05. To reduce the maximum force P_t and, at the same time, the difference $P_t - P_r$, the equidistant force application has been dropped. The mirror will be supported instead at non-equidistant points spaced more closely near the horizontal diameter, less closely near the vertical one. This device introduces unwanted Fourier modes $n = 2, 3, 4\ldots$ which are eliminated by active control with the above-mentioned small correction forces.

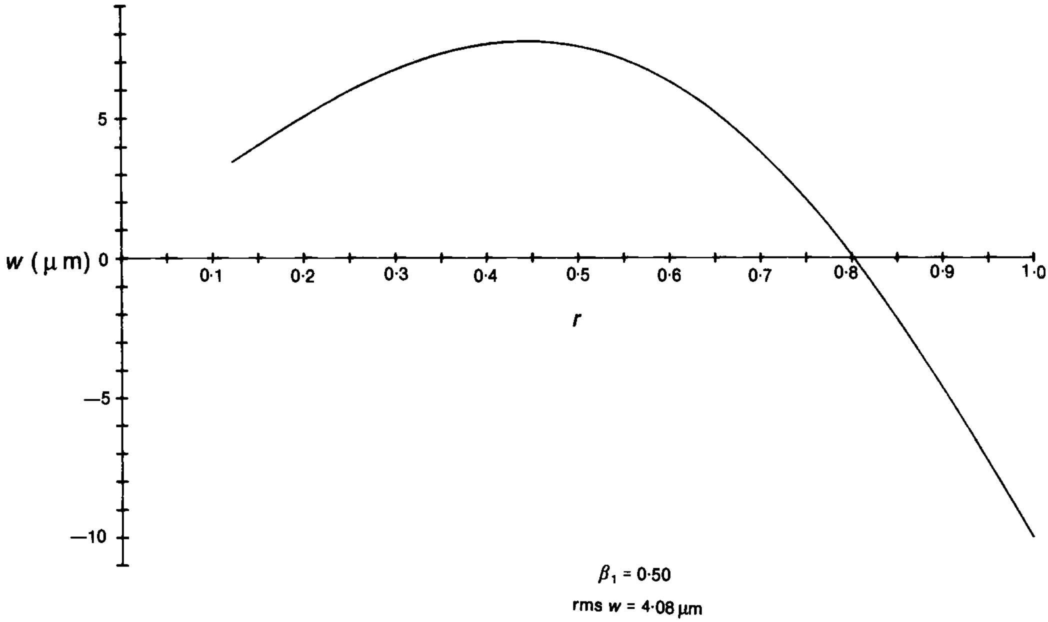


Figure 4. Deflection w of an 8-m meniscus laterally supported at the outer edge alone by standard push–pull–shear ($\beta_1 = 0{\cdot}50$) and balanced by axial edge forces V_1 only.

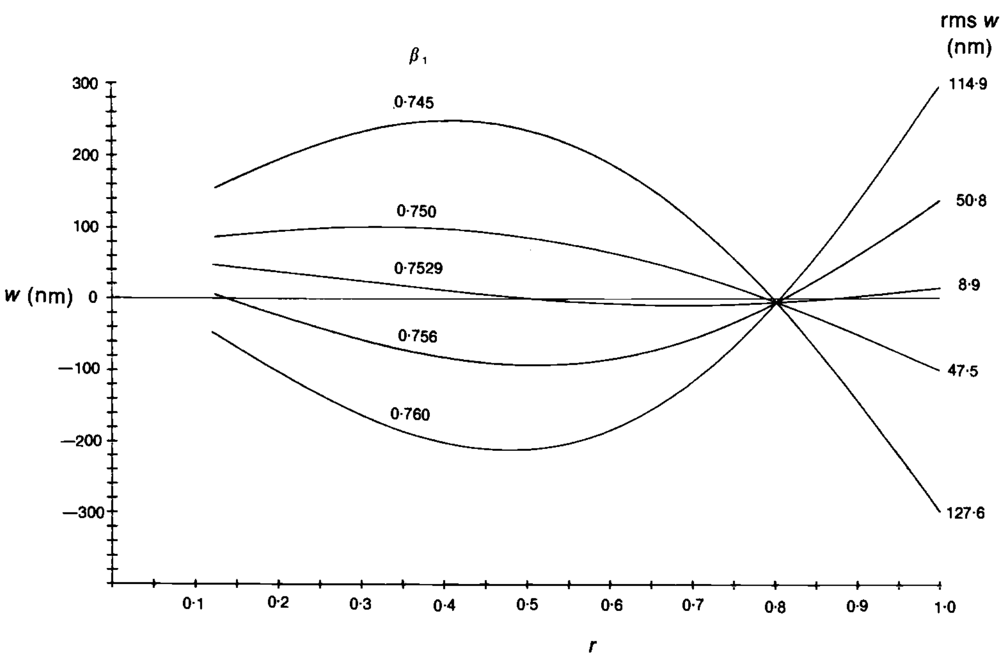


Figure 5. Deflections w of an 8-m meniscus supported as in figure 4, but with shear fractions β_1 increased to values near 0·75.

4.2. *Partial weight support and balancing forces at the hole*

The result just discussed is so good that there is hardly any room for further improvement by varying the other parameters which have so far been put equal to zero. Nevertheless, the investigation did include such possibilities—namely, the effect of simultaneous axial forces at the hole, of bending moments at the outer edge without and with axial forces at the hole, of a partial weight support at the hole with balancing axial forces at the outer edge and at both edges, of partial weight support at the hole with bending moments at both edges, and, finally, of the preceding option in conjunction with axial forces at the hole.

All these attempts failed to yield further improvement or, at least, to effect a worthwhile reduction of the parameter values β_1, which would have been a practical advantage. Only the second-best case, the last option on the listing above, will finally be presented although it is already much inferior to the first solution. The parameters were chosen as follows

$$\beta_0 = 0{\cdot}5,$$

$$\left.\begin{aligned} \sigma &= 0{\cdot}2, \\ t_0 &= -20\,\text{mm}, \\ t_1 &= 20\,\text{mm}, \end{aligned}\right\} \quad \text{specified as extreme values by ESO,}$$

$$\beta_1 = 0{\cdot}71.$$

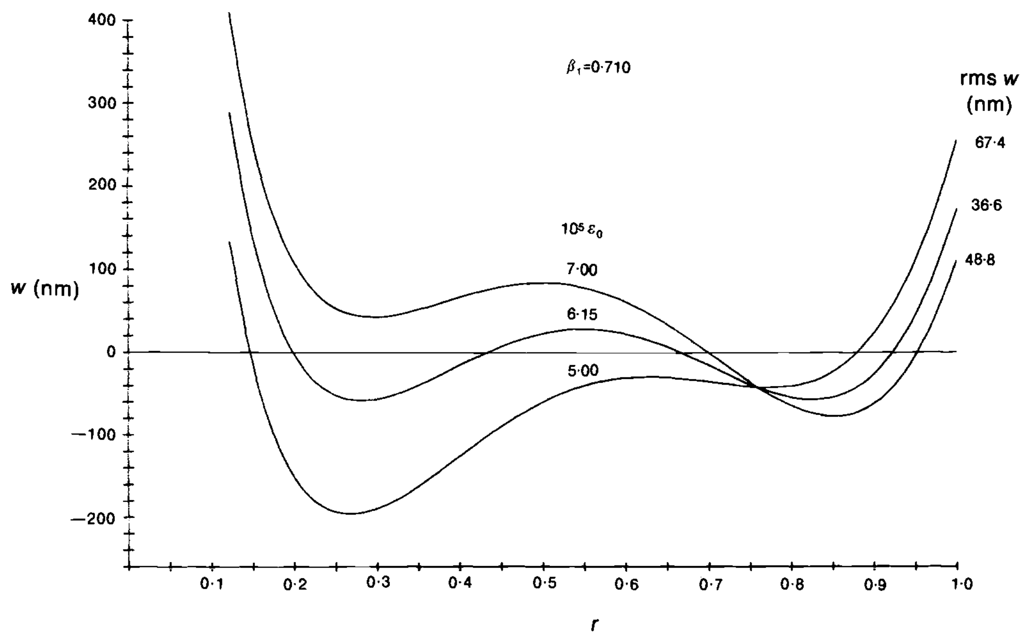


Figure 6. Deflections w of an 8-m meniscus supported at both edges: 20% of its weight is supported at the inner edge with a shear fraction $\beta_0 = 0{\cdot}50$. The shear fraction β_1 at the outer edge $= 0{\cdot}71$. Balancing moments are applied at both edges, equivalent to support force displacements $t_0 = -20$ mm and $t_1 = +20$ mm. The balancing axial forces at the outer edge are further reduced by assisting axial forces at the inner edge, corresponding to different values of ε_0.

The right-hand sides of all boundary conditions then take non-zero values. Three values were tried for the parameter ε_0 which determines the axial force V_0 in the second condition. The third condition changes to $P_{t_0}/P_{r_0}=1$.

Figure 6 shows the corresponding deflection curves. The bending moments now applied at both edges, equivalent to the displacements t_0 and t_1, act in such directions that back up the balance of the meniscus. Smaller axial forces at the outer edge therefore suffice to maintain equilibrium. Nevertheless, the rms deflection undergoes a four-fold increase relative to the first case. This is obviously due to the steep gradients of the deflections at the edges. Thus, edge moments may do more harm than axial forces otherwise required for reasons of equilibrium, as was the case in the first example. This conclusion as well as the demonstrated beneficial effect of tangential shear does not seem to agree with a statement in [10].

It must be pointed out that the second example has not yet been fully optimized so that some margin for further improvement may still exist. However, it is virtually certain that even an optimized solution would never match the low deflection values of the first case, apart from the fact that it would still suffer from pronounced higher orders of coma, as mentioned at the beginning and as borne out by the character of the curves in figure 6. A very slight advantage would only exist with regard to the smaller value β_1.

5. Outlook

As a sequel to the reported results, an interesting question remains to be answered: How can the design of lateral supports for large structured mirrors profit from the possibilities here derived for solid mirrors?

Acknowledgments

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